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This report summarizes the publications from our research on methods for analyzing stochastic systems. We studied three different system classes: (a) Probabilistic networks that model a variety of industrial and communications systems. These systems include data communications networks, voice communications networks, transportation networks, computer architectures, and electrical power systems. We corrected existing algorithms, derived the computational complexity of certain evaluations, and, based on new theoretical results, we proposed generalized algorithms that compute a performability measure by means of an iterative partition of the network state space. We also developed confidence intervals for Monte Carlo simulations tailored to the estimation of performability measures. (b) "Intelligent" Markovian networks where the processing of the units at the nodes and the routing of the units depend dynamically on the network congestion, and units can move concurrently. (c) Highly dependable systems with repairs. We have identified problems with existing simulation methods for estimating dependability measures and we are currently developing new methods that appear to be successful in a variety of large systems.

Progress Report
Final Report
AFOSR Grant F49620–93–1–0043
A Class of Methods for Analyzing Stochastic Systems

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This document is the last progress report and final report for grant F49620–93–1–0043. My research has focused on three different areas, and has resulted in eight papers and two completed doctoral dissertations. Furthermore, one doctoral student is currently working under my supervision on problems that should be of great interest to Air Force laboratories and the airline industry.

The following sections describe my contributions in each research area during the last three years.

During the last year (8/1/94 to 7/31/95), I worked on papers 2, 5, 6, 7, 8 described in Sections 1 and 2, and I advised a doctoral student on the topic described in Section 3.

1 Probabilistic Networks

Probabilistic networks are used to model a variety of industrial and communications systems. These systems include data communications networks, voice communications networks, transportation networks, computer architectures, and electrical power systems. Stochastic networks are modeled by graphs in which each arc, and probably each node, is assigned a nonnegative random weight. The component weights have interpretations depending on the type of network under consideration. My research has focused on the evaluation of general *performability* measures and considered the following types of systems:

Flow Networks The nodes model distribution centers and the arcs represent the means of transmitting commodities between pairs. The nodes are classified into sources, demand nodes, and transshipment nodes. The weight on each arc and transshipment node represents a capacity that limits the total amount of commodity that can be transmitted. An arc may also be associated with a random cost per unit of transmitted commodity.

The following are typical measures of interest: (a) The probability that the demands can be satisfied; (b) The probability that a given set of links and nodes limits commodity transmission when the demands cannot be satisfied; (c) The expected amount of unsupplied flow when the demands cannot be satisfied; (d) The probability that the total cost for satisfying the demands does not exceed a specified value.

Transportation Networks The arcs represent sections of routes and the nodes represent intersections of routes. The weight of an arc represents its length or travel time. A list of interesting problems includes the computation of: (a) The distribution of the shortest path length from a source s to a destination t; (b) The probability that a given arc belongs to a shortest path.

Undirected Networks Networks with undirected arcs are often used for modeling communications systems or for solving a variety of problems. An example is a graph whose

arcs have random costs and the objective is the evaluation of the probability that the nodes can be connected via a spanning tree whose total cost does not exceed a given budget.

The majority of problems for computing performability measures for stochastic networks are *#P-hard.* This property has motivated the research for approximation methods (see [2] for a comprehensive review of the relative literature). One class of these methods attempts to compute bounds while another class focuses on Monte Carlo estimation methods.

My research has focused on a methodology that are based on iteration and, in short, evaluate a performability measure as follows: At each iteration, a subset of the system state space is partitioned into sets with known contribution to the measure, sets with zero contribution, and *undetermined* sets whose value is unknown. The method continues in the same fashion until no undetermined sets remain. The proposed methods have the following important properties:

- After each iteration, they produce lower and upper bounds that improve continuously.

- The bounds along with the remaining undetermined sets can be used for designing Monte Carlo sampling plans that (a) yield estimates with variance smaller by several orders of magnitude than the variance of the respective estimates produced by a crude Monte Carlo experiment with equal sample size and (b) take less time than the crude experiment.

The following is a summary on the papers in this area.

1. A Note on State-Space Decomposition Methods for Analyzing Stochastic Flow Networks by the PI. *IEEE Transactions on Reliability.* 44(2), 354–357, 1995.

Consider a flow network with single source s and single sink t with demand $d > 0$. Assume that the nodes do not restrict flow transmission and the arcs have finite random discrete capacities. This paper has two objectives: (1) It corrects errors in well-known algorithms by Doulliez and Jamoulle [1] for (a) computing the probability that the demand is satisfied (or network reliability), (b) the probability that an arc belongs to a minimum cut which limits the flow below d, and (c) the probability that a cut limits the flow below d; (2) It discusses the applicability of these procedures.

The D&J algorithms are frequently referenced or used by researchers in the areas of power and communication systems and appear to be very effective for the computation of the network reliability when the demand is close to the largest possible maximum flow value. Extensive testing is required before the D&J algorithms are disposed in favor of

alternative approaches. Such testing should compare the performance of existing methods in a variety of networks including grid networks and dense networks of various sizes.

2. State Space Partitioning Methods for Stochastic Shortest Path Problems by the PI. To appear in *Networks*.

This paper describes methods for computing measures related to shortest paths in networks with discrete random arc lengths. These measures include the probability that there exists a path with length not exceeding a specified value and the probability that a given path is shortest. The proposed methods are based on an iterative partition of the network state space and provide bounds that improve after each iteration and eventually become equal to the respective measure. These bounds can also be used for constructing simple variance reducing Monte Carlo sampling plans, making the proposed algorithms useful for large problems where exact algorithms are virtually impossible. The proposed approach differs from existing approaches in that it attempts to derive "optimal" partitions. The algorithms can be easily modified to compute performance characteristics of stochastic activity networks. Computational experience has been encouraging as we have been able to solve networks that have presented problems to existing methods.

3. State Space Decomposition Methods for Solving a Class of Stochastic Network Problems by Jacobson, J. A. PhD Dissertation, Georgia Institute of Technology, 1993.

This study focuses on state space partitioning techniques for computing measures related to the operation of stochastic systems. These methods iteratively decompose the system state space until the measure of interest has been determined. The information available in each iteration yields lower and upper bounds on this measure, and can be used to design efficient Monte Carlo estimation routines. We present here new theoretical results identifying strategies for significantly enhancing the performance of these algorithms. Using these results, we describe a generalized algorithm that can easily be tailored to address a variety of problems. We next use this algorithm to analyze two important models in the area of stochastic network optimization.

The first model concerns the probabilistic behavior of minimum spanning trees in graphs with discrete random arc weights. Specifically, we compute the probability distribution of the weight of a minimum spanning tree and the probability that a given arc is on a minimum spanning tree. Both of these problems are shown to be #P-hard but the *matroidal* structure of the minimum spanning tree problem gives rise to an impressive algorithm for computing the probability that an arc belongs to a minimum spanning tree.

The second model considers minimum cost flows in networks with discrete random arc costs and capacities. We consider the case of statistically independent costs and capacities for each arc as well as the case in which the cost and capacity of each arc

change simultaneously. In each case, we show that the evaluation of the distribution of the minimum cost flow for a fixed demand configuration is a #P-hard problem. Numerical examples are given throughout the thesis.

Overall, this thesis makes the following contributions:

- Advances the understanding of state space partitioning methods. In doing so, it makes these methods more accessible and draws strong conclusions about the performance of certain types of partitions.
- It proposes areas in which further gains can be made with regards to these powerful computational techniques.

We have written a lengthy paper that is going to be published in a special issue on reliability of an archival journal. A second paper is in the final processing stage.

4. Distribution-free Confidence Intervals for Conditional Probabilities and Ratios of Expectations by the PI. *Management Science*. 40(12), 1748–1763, 1994.

Many simulation experiments are concerned with the estimation of a ratio of two unknown means, the estimation of a conditional probability being an example. This paper proposes confidence intervals for the case in which the ratio is estimated by using independent, identically distributed random pairs with bounded and ordered components. Emphasis is given to the case in which each component can be expressed as the product of a Bernoulli and a bounded random variable. The proposed intervals result from distribution-free, Bernstein-type bounds on error probabilities, are valid for every sample size, and their asymptotic width decreases at the same rate as that of confidence intervals based on the central limit theorem. Experimental results show that the proposed intervals are conservative with superior coverage for small sample sizes (≤ 50). This superiority over "normal" confidence intervals makes them useful for Monte Carlo experiments for estimating performability measures of probabilistic networks.

5. Conservative Confidence Intervals for Multinomial Probabilities by the PI and A. F. Seila. To appear in *Operations Research Letters*.

Multinomial data are often produced as a result of survey sampling where questions may be answered by selecting one of a set of mutually exclusive choices. For example, suppose that a system has $k-1$ mutually exclusive failure modes and cell i represents the event that the system fails according to mode i in a specific time period. The event that the system does not fail is represented by cell k. A simulation run of n independent replications will produce multinomial data providing the number of replications in which the system did not fail, or failed according to each failure mode.

This paper proposes distribution-free confidence intervals for multinomial experiments. Below, we briefly discuss the single, but important, result of this paper. Let $p = (p_1, p_2, \ldots, p_k)$ denote the unknown cell probabilities and suppose that we draw n samples. Let $n = (n_1, n_2, \ldots, n_k)$ be the observed counts and denote the observed cell proportions by $\hat{p}_i = n_i/n, i = 1, \ldots, k$.

The proposed confidence intervals have the form

$$\hat{p}_i \pm t/\sqrt{n}, \quad i = 1, \ldots, k$$

with simultaneous confidence coefficient

$$\Pi(k, p; n, t) = P\left[\bigcap_{i=1}^{k} |\hat{p}_i - p_i| < t/\sqrt{n}\right] \geq 1 - \alpha, \quad \alpha \in (0, 1).$$

Our methodology is based on the bound

$$\Pi(k, p; n, t) \geq G(k, p; n, t) \geq 1 - 2 \sup_p G(k, p; n, t),$$

where

$$G(k, p; n, t) = \sum_{i=1}^{k} \exp\left\{-nt\left[\left(1 + \frac{p_i(1-p_i)\sqrt{n}}{t}\right)\ln\left(1 + \frac{p_i(1-p_i)\sqrt{n}}{t}\right) - 1\right]\right\},$$

and finds the smallest t such that

$$\sup_p G(k, p; n, t) = \alpha/2.$$

A lengthy proof shows that $G(k, p; n, t)$ is minimized when $p_1 = \cdots = p_m = 1/m$ for some $2 \leq m \leq k$ and $p_i = 0$ for $i > m$.

The following table summarizes our findings. The last column lists asymptotically valid "normal" confidence intervals from Fitzpatrick and Scott [4]. The entries in column 3 are valid for all $k \geq 3$. For example, when $n = 500$ the intervals $\hat{p}_i \pm 1.67/\sqrt{500} = \hat{p}_i \pm 0.075$ have joint coverage probability at least 0.95 regardless of the number of cells. The inflated width is consistent with expectations and seems a reasonable price to pay for robustness against the usual normality assumptions.

$1-\alpha$	n	t	*asymptotic normal*
0.90	50	1.53	$\hat{p}_i \pm 1.00/\sqrt{n}$
	100	1.48	
	200	1.44	
	500	1.41	
	1000	1.40	
	∞	1.36	
0.95	50	1.67	$\hat{p}_i \pm 1.13/\sqrt{n}$
	100	1.62	
	200	1.58	
	500	1.54	
	1000	1.53	
	∞	1.48	
0.95	50	1.99	$\hat{p}_i \pm 1.40/\sqrt{n}$
	100	1.92	
	200	1.87	
	500	1.82	
	1000	1.79	
	∞	1.73	

6. Minimal Connected Enclosures on an Embedded Planar Graph by the PI, J. S. Provan, H. D. Ratliff, and B. R. Stutzman. Submitted for publication, 1995.

The purpose of this paper is to develop algorithms for combining regions formed by embedded planar graphs. Planar graphs are used to represent many systems with transportation networks (e.g., roads, rivers, rail) being examples. There are a variety of sources including the U.S. government for such databases. In these networks, edges represent transportation links augmented with additional edges for natural boundaries (e.g., rivers), man-made boundaries (e.g., power lines), and political boundaries (e.g., county lines), and vertices are formed from the intersections of these elements. Our work is motivated by applications in the areas of network design, reliability, distribution and logistics, and geographic information systems.

We study five problems of finding minimal enclosures on a connected plane graph. The first three problems consider the identification of a shortest enclosing walk, cycle or trail surrounding a polygonal, simply connected obstacle on the plane. We propose polynomial algorithms that improve over existing algorithms. The last two problems consider the formation of minimal zones (sets of adjacent regions such that any pair of points in a zone can be connected by a non-zero width curve that lies entirely in the zone). Specifically, we assume that the regions of the graph have nonnegative weights and seek the formation of minimum weight zones containing a set of points or a set of

regions. We prove that the last two problems are NP-hard and transform them to Steiner arborescence/fixed-charge flow problems.

2 Markovian Network Processes

Markovian network processes have been used for describing the movement of parts and supplies in manufacturing and distribution systems as well as the movement of telephone calls and data packets in communications systems. The distinguishing feature of my research in this area with Richard Serfozo and Akram El-Tannir is the emphasis on the next generation of "intelligent" networks where the processing of the units at the nodes and the routing of the units depend dynamically on the state of the network, and units move concurrently (as with batch processing).

Most of the existing theory on Markovian network processes is for networks in which the units operate independently and move one-at-a-time, and their routes are independent. Our goal is to enhance the understanding of those complex networks by describing their stochastic behavior.

My two joint publications in this area are listed below.

7. A Multivariate Generalization of Markov Modulated Processes by the PI, A. El-Tannir, and R. F. Serfozo. Submitted for publication, 1995.

Markov modulated processes model queueing systems where the arrival and service rates vary according to a Markov process independently of the number of customers in the system. These processes, however, do not cover systems where the arrival and service rates depend on the number of customers present. An example is an $M/M/Y$ system where the number of servers $Y(t)$ at time t is a Markov process with rates that depend on the number of customers present.

This paper studies a family of multivariate Markov processes where transitions can take place simultaneously and the rate at which a set of components changes state depends on the state of the remaining components. This family covers a wide range of Markov processes including Markov modulated processes, Markovian queues with variable capacity, and standard network processes such as closed Jackson network processes. The paper makes the following two contributions: (a) It identifies processes whose stationary distributions have product form; (b) It presents approximations for stationary distributions. The main result proposes an approximation for a bivariate process (X, Y) based on an "auxiliary" process with "averaged" rates. When the component X has n states and the component Y has m states, the computation of the approximate distribution requires the solution of $m + 1$ subsystems each with dimensions $n \times n$ instead of solving an $(mn) \times (mn)$ system. We proceed by generalizing this result for multivariate processes, and conclude with additional approximations. We illustrate the proposed

techniques by analyzing the equilibrium behavior of several practical systems.

8. Partition-Balanced Markov Processes by the PI, A. El-Tannir, and R. F. Serfozo. Submitted for publication, 1995.

When can the stationary distribution of a Markov process be obtained by pasting together several stationary distributions that represent the process restricted to certain subspaces? This study describes a class of "partition-balanced" Markov processes that have this cut-and-paste or divide-and-conquer property. The importance of this property is that the problem of obtaining a stationary distribution on a large space (e.g., for networks) reduces to finding several stationary distributions on smaller subspaces, either by analytical means or simulations or by a combination of both.

The notion of partition-balance is a "macro-reversibility" property resembling the detailed balance property of reversible processes. We present several characterizations of partition-balance and identify subclasses of treelike, starlike and circular partition-balanced processes. A new circular birth-death process is used in the analysis. The results are illustrated by a queueing model with controlled service rate, a multi-type service system with blocking and a parallel-processing model. A few comments address partition-balance for non-Markovian processes.

3 Variance Reduction Methods for Simulating Highly Dependable Systems with Repairs

The development of methods for simulating highly dependable systems with repairs has been a popular research topic within the simulation and computer science communities during the last decade. Since failures in such systems are rare events, the estimation of system dependability measures such as the limiting (long run) unavailability and the mean time to failure require prohibitively long simulation runs.

A variety of papers (see [5] and [6]) have developed variance reducing techniques that use the importance sampling method. Specifically, those papers propose the combination of importance sampling with the regenerative method for estimating long run measures, and the combination of importance sampling with the conditional Monte Carlo method for estimating transient measures such as the average interval availability or the distribution of the interval availability.

Bruce Shultes, a doctoral student whose research has been funded by this grant during the last two years, attempted to improve on existing methods (e.g., *failure biasing* [5, 6] and *failure distance biasing* [3]) by using structural network information such as the state of a cut vector or a path vector. To our surprise, the existing methods failed to induce substantial variance reduction over the crude Monte Carlo method in networks

with complex structure. In several cases, they produced inflated variance estimates.

The following list contains our conclusions and results during the last two years.

- The existing methods are geared towards short paths to failure. Hence they have problems in systems with large cuts.
- The applicability of existing algorithms is limited to systems where each individual component is highly dependable. This limitation excludes systems whose dependability is due to redundancies at the component level.
- There exist near optimal importance sampling distributions that are non-stationary and appear to resolve the aforementioned problems.
- Several stationary importance sampling distributions that appear to perform better than existing methods have been identified.

Bruce Shultes will graduate by the Summer of 1996. The contribution of this grant to his academic achievements will be acknowledged in his dissertation as well as in the subsequent publications on this topic.

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